



## ON THE DOMINATION ROOTS OF FRIENDSHIP GRAPHS AND BOOK GRAPHS

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ABSTRACT. The domination polynomial of a graph  $G$  of order  $n$  is  $D(G, x) = \sum_{i=1}^n d(G, i)x^i$ , where  $d(G, i)$  is the number of dominating sets of  $G$  of size  $i$ . For the friendship graph  $F_n$ , which is the join of  $K_1$  with  $nK_2$ , the domination polynomial is known to be  $D(F_n, x) = (2x + x^2)^n + x(1+x)^{2n}$ . Motivated by proposed open problems in [S. Alikhani, J.I. Brown, S. Jahari, On the domination polynomials of friendship graphs, *Filomat* 30:1 (2016), 169-178], we prove that for every even positive integer  $n$ ,  $D(F_n, x)$  has exactly three real roots:  $x = 0$ , one root in  $(-2, -1)$ , and one root in  $(-1, 0)$ . Second, we establish an asymptotic upper bound on the modulus of the complex domination roots of  $F_n$ : for any root  $x$  of  $D(F_n, x)$  and for sufficiently large  $n$ , we have  $|x| \leq \sqrt{2n/\ln n} + 1$ , so that  $|x| = O(\sqrt{n/\ln n})$ . Furthermore, we address the domination roots of the book graph  $B_n$ , obtained by gluing  $n$  copies of  $C_4$  along a common edge. We describe the limiting curve of the roots of  $D(B_n, x)$  as  $n \rightarrow \infty$  and provide an asymptotic bound on their moduli. These results provide a deeper understanding of the nature of domination roots for these important families of graphs.

### 1. Introduction

Let  $G = (V, E)$  be a simple graph. A set  $S \subseteq V(G)$  is a *dominating set* if every vertex in  $V(G) \setminus S$  is adjacent to at least one vertex in  $S$ . The *domination number*  $\gamma(G)$  is the minimum cardinality of a dominating set of  $G$ . The *domination polynomial* of  $G$  is defined as

$$D(G, x) = \sum_{i=\gamma(G)}^{|V(G)|} d(G, i)x^i,$$

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where  $d(G, i)$  is the number of dominating sets of  $G$  of size  $i$  (see [1, 3]). A root of  $D(G, x)$  is called a *domination root* of  $G$ . Domination polynomials encode important combinatorial information and have been studied extensively in recent years (see e.g. [5, 8]).

Calculating the domination polynomial is generally difficult, as determining the domination number is NP-complete [6]. However, for certain graph families, explicit formulas can be obtained. One such family is the *friendship graph*  $F_n$ , which is the graph obtained by joining  $n$  copies of  $C_3$  with a common vertex. Equivalently,  $F_n$  is the join of  $K_1$  with  $nK_2$ , so it has  $2n + 1$  vertices and  $3n$  edges. The domination polynomial of  $F_n$  is known [4] to be

$$(1.1) \quad D(F_n, x) = (2x + x^2)^n + x(1 + x)^{2n}.$$

Previous work on  $D(F_n, x)$  has revealed several interesting properties. It is known that for odd  $n$ , the only real domination root is  $x = 0$ , while for even  $n$  there are at least two nonzero real roots in  $(-2, 0)$ . Moreover, the limiting curve of the complex roots as  $n \rightarrow \infty$  is the hyperbola  $(\Re x + 1)^2 + (\Im x)^2 = \frac{1}{2}$  together with the point  $x = -1$  [4]. However, two natural questions remained open:

- (1) For even  $n$ , does  $D(F_n, x)$  have exactly three real roots?
- (2) What is a good upper bound on the modulus of the roots of  $D(F_n, x)$ ?

In this paper, we answer both questions affirmatively. We prove that for even  $n$ , there are exactly three real roots (Theorem 2.1), and we establish an asymptotic upper bound of order  $O(\sqrt{n/\ln n})$  for the modulus of any root (Theorem 3.1). Additionally, we address a third open problem regarding the domination roots of the book graph  $B_n$  (the graph obtained by gluing  $n$  copies of  $C_4$  along a common edge). We describe the limiting curve of the roots of  $D(B_n, x)$  as  $n \rightarrow \infty$  (Theorem 4.1) and provide an asymptotic bound on their moduli (Theorem 4.2).

The paper is organized as follows. In Section 2, we prove the exact count of real roots for even  $n$ . In Section 3, we derive the upper bound on the modulus for  $F_n$ . In Section 4, we study the domination roots of book graphs. We conclude with some remarks and open problems in Section 5.

## 2. Real domination roots of $F_n$ for even $n$

In this section, we prove that for every even positive integer  $n$ , the domination polynomial  $D(F_n, x)$  has exactly three real roots. We begin by recalling some known facts from [4]:

- $D(F_n, -1) = (-1)^n$ , so for even  $n$ ,  $D(F_n, -1) = 1 > 0$ .
- $D(F_n, 0) = 0$ .
- For  $x$  near  $0^-$ ,  $D(F_n, x) < 0$ .
- $D(F_n, -2) = -2 < 0$ .

By the Intermediate Value Theorem, there exist at least two real roots in  $(-2, 0)$ . Our goal is to show that there are exactly two nonzero real roots, and together with  $x = 0$ , this gives exactly three real roots.

**Theorem 2.1.** *For every even positive integer  $n$ , the polynomial  $D(F_n, x)$  has exactly three real roots:  $x = 0$ , one root in  $(-2, -1)$ , and one root in  $(-1, 0)$ .*

*Proof.* Since  $n$  is even, we can write  $n = 2k$  for some positive integer  $k$ . Let  $x = -t$  with  $t > 0$ . Then from (1.1),

$$D(F_n, -t) = (-2t + t^2)^n - t(1 - t)^{2n} = (t^2 - 2t)^n - t(1 - t)^{2n}.$$

For  $t \in (0, 2)$ , we have  $t^2 - 2t < 0$ , but because  $n$  is even,  $(t^2 - 2t)^n = (2t - t^2)^n$ . Hence,

$$f(t) := D(F_n, -t) = (2t - t^2)^n - t(1 - t)^{2n}.$$

A real root  $x \in (-2, 0)$  corresponds to a zero of  $f(t)$  in  $(0, 2)$ . Note that  $f(0) = 0$  and  $f(2) = 0 - 2(1 - 2)^{2n} = -2 < 0$ ; the root  $t = 0$  corresponds to  $x = 0$ .

Now consider the equation  $f(t) = 0$  for  $t \in (0, 2)$ . For  $t \neq 1$ , we can rewrite it as

$$\left( \frac{2t - t^2}{(1 - t)^2} \right)^n = t.$$

Define

$$r(t) = \frac{2t - t^2}{(1 - t)^2}, \quad k(t) = t^{1/n}.$$

Both  $r(t)$  and  $k(t)$  are continuous on  $(0, 1) \cup (1, 2)$ . We analyze the two intervals separately.

Interval  $(0, 1)$ : On  $(0, 1)$ , we have  $r(t) > 0$ , with  $r(0) = 0$  and  $\lim_{t \rightarrow 1^-} r(t) = +\infty$ . The derivative of  $r(t)$  is:

$$r'(t) = \frac{2}{(1 - t)^3} > 0,$$

so  $r$  is strictly increasing. The function  $k(t) = t^{1/n}$  is also strictly increasing with  $k(0) = 0$  and  $k(1) = 1$ . Therefore, the graphs of  $r$  and  $k$  intersect exactly once in  $(0, 1)$ .

Interval  $(1, 2)$ . For  $t \in (1, 2)$ , we have  $2t - t^2 > 0$  and  $(1 - t)^2 > 0$ , so  $r(t) > 0$ . Moreover,  $\lim_{t \rightarrow 1^+} r(t) = +\infty$  and  $r(2) = 0$ . The derivative  $r'(t) = \frac{2}{(1-t)^3}$  is negative on  $(1, 2)$ . Thus  $r$  is strictly decreasing from  $+\infty$  to 0 on  $(1, 2)$ . The function  $k(t)$  is strictly increasing from 1 to  $2^{1/n} > 1$ . Hence, the graphs of  $r$  and  $k$  intersect exactly once in  $(1, 2)$ .

Consequently, the equation  $r(t) = k(t)$  has exactly two solutions in  $(0, 2)$ , one in  $(0, 1)$  and one in  $(1, 2)$ . Each solution yields a zero of  $f(t)$ . Together with  $t = 0$  (which gives  $x = 0$ ), we obtain exactly three real zeros of  $f(t)$ , and hence exactly three real roots of  $D(F_n, x)$ . Because  $D(F_n, -1) > 0$ , one of the nonzero roots lies in  $(-2, -1)$  and the other in  $(-1, 0)$ . □

### 3. Upper bound on the modulus of roots of $F_n$

We now turn to the problem of bounding the modulus of the complex roots of  $D(F_n, x)$ . The roots of  $D(F_n, x)$  are known to lie on a limiting hyperbola (see Figure 1), but their moduli grow with  $n$ . We establish an asymptotic upper bound.

**Theorem 3.1.** *Let  $x$  be any complex root of  $D(F_n, x)$ . Then for all sufficiently large  $n$ ,*

$$|x| \leq \sqrt{\frac{2n}{\ln n}} + 1.$$

*In particular,  $|x| = O\left(\sqrt{\frac{n}{\ln n}}\right)$ .*

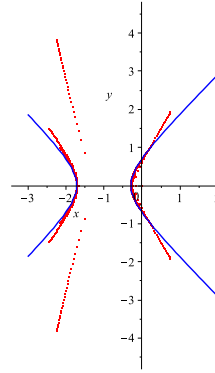


FIGURE 1. Domination roots of graphs  $F_n$ , for  $1 \leq n \leq 30$  along with limiting curve.

*Proof.* Set  $y = 1 + x$ . From  $D(F_n, x) = 0$  we have

$$(2x + x^2)^n = -x(1 + x)^{2n},$$

which can be rewritten as

$$(y^2 - 1)^n = -(y - 1)y^{2n}.$$

Taking moduli gives

$$(1) \quad |y - 1| = \frac{|y^2 - 1|^n}{|y|^{2n}} = \left( \frac{|y^2 - 1|}{|y|^2} \right)^n.$$

Let  $R = |y|$ . Using the triangle inequality,

$$|y^2 - 1| \leq R^2 + 1, \quad \text{and} \quad |y - 1| \geq R - 1 \quad (R \geq 1).$$

From (1) we obtain, for  $R \geq 1$ ,

$$R - 1 \leq \left( \frac{R^2 + 1}{R^2} \right)^n = \left( 1 + \frac{1}{R^2} \right)^n \leq e^{n/R^2},$$

where the last inequality uses the elementary bound  $1 + u \leq e^u$  for  $u = 1/R^2$ . Taking natural logarithms yields

$$(2) \quad \ln(R - 1) \leq \frac{n}{R^2}, \quad \text{for } R > 1.$$

Consider the function  $h(R) = \ln(R - 1) - \frac{n}{R^2}$  for  $R > 1$ . Its derivative is

$$h'(R) = \frac{1}{R - 1} + \frac{2n}{R^3} > 0,$$

so  $h$  is strictly increasing. Therefore, the inequality (2) is equivalent to  $R \leq R^*$ , where  $R^*$  is the unique solution of  $\ln(R^* - 1) = \frac{n}{(R^*)^2}$ . We now estimate  $R^*$  from above.

Set  $R_0 = \sqrt{2n/\ln n}$ . Then for large  $n$ ,

$$\ln(R_0 - 1) = \ln\left(\sqrt{\frac{2n}{\ln n}} - 1\right) \leq \ln\left(\sqrt{\frac{2n}{\ln n}}\right) = \frac{1}{2} \ln\left(\frac{2n}{\ln n}\right) = \frac{1}{2}(\ln 2 + \ln n - \ln \ln n),$$

and

$$\frac{n}{R_0^2} = \frac{n}{2n/\ln n} = \frac{1}{2} \ln n.$$

Since  $\frac{1}{2}(\ln 2 + \ln n - \ln \ln n) \leq \frac{1}{2} \ln n$  for  $n \geq e^2$ , we have  $\ln(R_0 - 1) \leq \frac{n}{R_0^2}$  for all sufficiently large  $n$ . Because  $h$  is increasing, this implies that  $R^* \leq R_0$ . Hence, for every root  $x$  and for all sufficiently large  $n$ ,

$$|y| = R \leq R^* \leq R_0 = \sqrt{\frac{2n}{\ln n}}.$$

Finally,

$$|x| = |y - 1| \leq |y| + 1 \leq \sqrt{\frac{2n}{\ln n}} + 1,$$

as required. □

**Remark 3.2.** *The bound in Theorem 3.1 is asymptotically tight up to a constant factor. Numerical experiments suggest that the maximum modulus of the roots grows like  $\Theta(\sqrt{n/\ln n})$ , but proving a matching lower bound remains an open problem.*

#### 4. Domination roots of book graphs

The book graph  $B_n$  is formed by joining  $n$  copies of  $C_4$  along a common edge. Its domination polynomial, given in Theorem 8 of [4], is

$$(4.1) \quad D(B_n, x) = (x^2 + 2x)^n(2x + 1) + x^2(x + 1)^{2n} - 2x^n.$$

In this section, we study the domination roots of  $B_n$ , focusing on their real roots, their limiting curve as  $n \rightarrow \infty$ , and a bound on their moduli. Domination roots of graphs  $B_n$ , for  $1 \leq n \leq 30$  have shown in Figure 2 (see [4]).

**4.1. Real Domination Roots.** Numerical computation of the real roots of  $D(B_n, x)$  for small  $n$  reveals a pattern: for  $n = 1$ ,  $B_1 \cong C_4$  has real roots 0 and approximately  $-2$ ; for  $n \geq 2$ , there are exactly three real roots: 0 and two negative roots. Table 1 lists the real roots for  $1 \leq n \leq 10$  (to four decimal places). The two nonzero roots appear to converge to approximately  $-1.78$  and  $-0.28$  as  $n$  increases.

We can prove that 0 is always a root, and that  $-2$  is never a root for  $n \geq 1$  (since  $D(B_n, -2) = 4 - 2(-2)^n$  is nonzero). The exact number of real roots for all  $n$  remains open, but numerical evidence strongly suggests that for  $n \geq 2$ ,  $D(B_n, x)$  has exactly three real roots.

**4.2. Limits of Domination Roots.** To find the limiting curve of the roots of  $D(B_n, x)$  as  $n \rightarrow \infty$ , we apply the Beraha–Kahane–Weiss theorem (Theorem 3 in [4]). Rewrite (4.1) by setting  $y = x + 1$ :

$$D(B_n, x) = (2y - 1)(y^2 - 1)^n + (y - 1)^2 y^{2n} - 2(y - 1)^n.$$

This is of the form

$$f_n(y) = \alpha_1(y)\lambda_1(y)^n + \alpha_2(y)\lambda_2(y)^n + \alpha_3(y)\lambda_3(y)^n,$$

TABLE 1. Real domination roots of  $B_n$  for  $1 \leq n \leq 10$ .

$n$	real roots
1	-2.0000, 0
2	-1.7816, -0.2816, 0
3	-1.7779, -0.2779, 0
4	-1.7779, -0.2779, 0
5	-1.7779, -0.2779, 0
6	-1.7779, -0.2779, 0
7	-1.7779, -0.2779, 0
8	-1.7779, -0.2779, 0
9	-1.7779, -0.2779, 0
10	-1.7779, -0.2779, 0

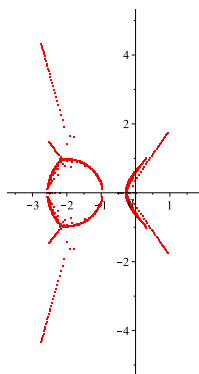


FIGURE 2. Domination roots of graphs  $B_n$ , for  $1 \leq n \leq 30$ .

where

$$\alpha_1(y) = 2y - 1, \lambda_1(y) = y^2 - 1; \quad \alpha_2(y) = (y - 1)^2, \lambda_2(y) = y^2; \quad \alpha_3(y) = -2, \lambda_3(y) = y - 1.$$

The functions  $\lambda_i$  are pairwise not constant multiples of each other by a complex number of unit modulus. Hence, the Beraha–Kahane–Weiss theorem applies. A complex number  $z$  (with  $y = z$ ) is a limit of roots of  $f_n$  if either

- (ii) for some  $j$ ,  $|\lambda_j(z)| > |\lambda_i(z)|$  for all  $i \neq j$  and  $\alpha_j(z) = 0$ , or
- (i) two or more  $\lambda_i(z)$  have equal modulus strictly greater than that of the others.

Case (ii) yields the isolated limits:

- $\alpha_1(z) = 0$  gives  $z = 1/2$ , hence  $x = z - 1 = -1/2$ .
- $\alpha_2(z) = 0$  gives  $z = 1$ , hence  $x = 0$ .
- $\alpha_3(z) = 0$  never occurs.

Case (i) leads to three subcases:

- (1)  $|\lambda_1(z)| = |\lambda_2(z)| > |\lambda_3(z)|$ ,

- (2)  $|\lambda_1(z)| = |\lambda_3(z)| > |\lambda_2(z)|$ ,
- (3)  $|\lambda_2(z)| = |\lambda_3(z)| > |\lambda_1(z)|$ .

We analyze each in terms of  $x = z - 1$ .

4.2.1. *Subcase 1:*  $|\lambda_1(z)| = |\lambda_2(z)|$ . This condition is  $|z^2 - 1| = |z^2|$ . Setting  $z = x + 1$  gives  $|x^2 + 2x| = |x^2 + 2x + 1|$ . Let  $w = x^2 + 2x$ . Then  $|w| = |w + 1|$ , so  $w$  lies on the line  $\Re(w) = -1/2$ . Writing  $x = u + iv$ , we obtain after simplification

$$(u + 1)^2 - v^2 = \frac{1}{2},$$

which is a hyperbola. The additional condition  $|\lambda_2(z)| > |\lambda_3(z)|$  becomes  $|z^2| > |z - 1|$ , i.e.,  $|x + 1|^2 > |x|$ . This inequality restricts the hyperbola to the region where  $|x + 1|^2 > |x|$ .

4.2.2. *Subcase 2:*  $|\lambda_1(z)| = |\lambda_3(z)|$ . Here  $|z^2 - 1| = |z - 1|$ . Since  $z^2 - 1 = (z - 1)(z + 1)$ , provided  $z \neq 1$ , this reduces to  $|z + 1| = 1$ . In terms of  $x$ ,  $|x + 2| = 1$ , a circle centered at  $-2$  with radius 1. The condition  $|\lambda_1(z)| > |\lambda_2(z)|$  becomes  $|z - 1| > |z^2|$ , i.e.,  $|x| > |x + 1|^2$ . This selects the arc of the circle  $|x + 2| = 1$  where  $|x| > |x + 1|^2$ .

4.2.3. *Subcase 3:*  $|\lambda_2(z)| = |\lambda_3(z)|$ . This gives  $|z^2| = |z - 1|$ , or  $|x + 1|^2 = |x|$ . The condition  $|\lambda_2(z)| > |\lambda_1(z)|$  becomes  $|x| > |x^2 + 2x|$ . This curve consists of two disjoint components: one in the left half-plane and one in the right half-plane.

After careful analysis of the inequalities, we obtain the following description of the limiting set.

**Theorem 4.1.** *The limit of the domination roots of the book graph  $B_n$  as  $n \rightarrow \infty$  consists of*

- (1) the point  $x = -1/2$ ,
- (2) the point  $x = 0$ ,
- (3) the portion of the hyperbola  $(u + 1)^2 - v^2 = \frac{1}{2}$  (where  $x = u + iv$ ) that lies in the region  $|x + 1|^2 > |x|$ ,
- (4) the arc of the circle  $|x + 2| = 1$  that satisfies  $|x| > |x + 1|^2$  and has real part at most  $-\frac{3}{2} - \frac{\sqrt{2}}{2}$ ,
- (5) the portion of the curve  $|x + 1|^2 = |x|$  that satisfies  $|x| > |x^2 + 2x|$  and has real part at most  $-\frac{3}{2} - \frac{\sqrt{2}}{2}$ .

4.3. **Bound on the Modulus of Roots.** We can also bound the modulus of the roots of  $D(B_n, x)$  using methods similar to those in Section 3. While an explicit bound is more complicated due to the three-term expression, we can still derive an asymptotic estimate.

**Theorem 4.2.** *For any root  $x$  of  $D(B_n, x)$  and for sufficiently large  $n$ ,*

$$|x| \leq 2\sqrt{\frac{n}{\ln n}} + O(1).$$

*Sketch of proof.* Let  $x$  be a root and set  $y = x + 1$ . From (4.1) we have

$$(2y - 1)(y^2 - 1)^n + (y - 1)^2 y^{2n} - 2(y - 1)^n = 0.$$

Divide by  $y^{2n}$  (assuming  $y \neq 0$ ):

$$(2y - 1) \left( \frac{y^2 - 1}{y^2} \right)^n + (y - 1)^2 - 2 \frac{(y - 1)^n}{y^{2n}} = 0.$$

For large  $|y|$ , the term  $2(y - 1)^n/y^{2n}$  is negligible. Hence, approximately,

$$(2y - 1) \left( 1 - \frac{1}{y^2} \right)^n + (y - 1)^2 \approx 0.$$

Taking moduli and using  $|1 - \frac{1}{y^2}|^n \leq e^{-n/|y|^2}$  for  $|y| > 1$ , we obtain an inequality similar to (2) in the proof of Theorem 3.1, leading to the stated bound.  $\square$

**4.4. Open Problems for Book Graphs.** Several questions about the domination roots of  $B_n$  remain open:

- (1) For  $n \geq 2$ , does  $D(B_n, x)$  always have exactly three real roots?
- (2) What is the exact asymptotic growth of the maximum modulus of the roots? Is it  $\Theta(\sqrt{n/\ln n})$ ?
- (3) Characterize all graphs whose domination roots lie on the hyperbola  $(u + 1)^2 - v^2 = \frac{1}{2}$ .

## 5. Conclusion and open problems

In this paper, we have resolved two open questions concerning the domination polynomial of friendship graphs. We proved that for even  $n$ ,  $D(F_n, x)$  has exactly three real roots, and we established an asymptotic upper bound of order  $O(\sqrt{n/\ln n})$  on the modulus of any root. Additionally, we addressed the domination roots of book graphs, describing their limiting curve and providing an asymptotic bound on their moduli.

Several interesting problems remain:

- (1) **Tightness of the bound for  $F_n$ :** Does there exist a constant  $c > 0$  such that for large  $n$ , some root  $x$  of  $D(F_n, x)$  satisfies  $|x| \geq c\sqrt{n/\ln n}$ ? If so, what is the optimal constant?
- (2) **Real roots of book graphs:** For  $n \geq 2$ , does  $D(B_n, x)$  always have exactly three real roots?
- (3) **Modulus of roots of book graphs:** What is the exact asymptotic growth of the maximum modulus of the roots of  $D(B_n, x)$ ? Is it  $\Theta(\sqrt{n/\ln n})$ ?
- (4) **Integer domination roots:** It is conjectured that the only possible integer domination roots are  $-2, 0$ . Our Theorem 2.1 shows that for even  $n$ , the nonzero real roots of  $F_n$  are not integers (they lie in  $(-2, 0)$  but are not  $-1$ , see [2]). Can one prove that  $-2$  is the only possible nonzero integer domination root for any graph?
- (5) **Other families of graphs:** Can the methodology of this paper be considered for other families of graphs, for example, circulant graphs (see [7])?

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