



## A TRIBUTE TO PROF. O. A. S. KARAMZADEH

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*Dedicated to Prof. O. A. S. Karamzadeh*

ABSTRACT. It gives me a great pleasure to write this short note about my meetings with prof. Karamzadeh, during my school days and recalling some elementary mathematics that we discussed. Of course his mathematical contributions to algebra and topology and to the mathematics of our country, will be mentioned by the experts in his fields of interest. All dates given are according to the Iranian calendar.

My first acquaintance with Karamzadeh goes back to the last years of the forties, to be precise 1347, when I was at my last years of high school. At that time, he was a student of the mathematics department at Tehran university and he used to come to his city of birth Masjed Soleyman during the summer time. In this city high school students usually formed small groups and invited him to teach them mathematics so that when the schools open in autumn they gather strong back ground.

As a matter of fact, a few of my class mates formed a group and invited Karamzadeh to teach them algebra, geometry and number theory. I did not have the opportunity to attend the class, but at the final stage of the class one of my classmates informed me of the class and showed me his handwriting mathematical notes that was taught by Karamzadeh in the class. I was really excited because it was the first time I saw mathematical problems not covered by school text books. By the suggestion of a friend I decided to meet Karamzadeh in person and I did. We arranged for a rendezvous and met in the main street of our city, Masjed Solayman, which is also spelled Masjed-I-Solayman (briefly, called M.I.S.) and soon my friend joined us too, on time. We found that Karamzadeh likes to talk about

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mathematics and recalling interesting problems from his school days. I remember that a problem was first posed as follows: suppose that there are ten bags each containing the coins with the same weight. Each genuine coin weighs 10 grams, but only one bag contains fake coins, where each of the coins weighs 9 grams. The bags are large enough so that we can take as many coins as we need from each bag to weigh them together. Now the problem is to detect the bag containing the counterfeits by one weighing. We thought about the problem for a moment and after a minute or two we came to a correct solution, Karamzadeh gave us a pat on the back and encouraged us a lot (note, just put the bags in one row and take one coin from the first bag, two from the second one and similarly do the same with each bag, and finally take 10 coins from the last bag). Then weigh all these 55 coins together. If all the coins were genuine, i.e., 10 grams each, we should get the number 550, for the total weight. Hence we certainly get a smaller number than 550. Now if this number is one unit less the first bag contains the fake coins, and if it is two units less the second bag is the desired one and so on. This problem was of interest to Karamzadeh at that time, later when he was the leader of our selected IMO students he generalized it and presented it to them in one of his classes in a training camp (note, for this generalization, he did not give any fixed number for the weight of each coin in the bags but we may assume the weight of each coin is less than, 50 say, grams and we may also replace the 10 bags by any number of bags and the problem is to determine the weight of each coin in the bags by just one weighing, note each bag is full of the genuine coins with the same weight, but different bags might contain different coins, and we are allowed to take any number of coins from each bag if we need to do so and have a super electronic scale to work with). We discussed some other elementary problems, where they could be discussed while walking on the street, for example if there exist positive real numbers  $x, y, z$  such that  $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 2$ . He did not wait for our possible solutions, he said the first problem needed no background, just your ability of thinking in a right direction was sufficient but for this one you have to know some facts. In this particular case we may promptly say by AM-GM inequality there are not such positive numbers, for the product of these three positive fractions is 1 and therefore their sum must be greater than or equal to 3. I remember another problem we discussed was the Morleys triangle, but of course not on the street while waking. In this case, by his suggestion we went into an ice cream shop to have ice cream while discussing the triangle, because we could draw configurations while sitting at the table. In the shop we insisted that we should have Dutch treat, but he did not give in and treated us by saying that I should pay for your solution of that coins problem, while laughing, and this ice cream is your rewards. As for, Morley's triangle, he just stated the problem, however he said even expert mathematicians cannot give a solution off-hand to this problem, and I do not expect you to be able to solve it here, unless you have already seen a solution before. Perhaps it was this experience that I later, as you will see briefly, have also dealt with Morley's Theorem. Now, we may recall that if in a triangle each angle is trisected, then the three points of intersection of the adjacent trisectors form an equilateral triangle called Morleys triangle. This triangle was discovered by Morley, an Anglo-American mathematician, around 1899. The discovery was called Morley's Trisector Theorem. Although the theorem has been proved by many authors it is called a magic theorem or a mysterious theorem by some of these authors. Because some of the proofs are very technical and

non-motivated and some are backwards and non-explanatory too or are non-geometrical. Some first-rate mathematicians like J. Conway, Alain Conne, and some other notable mathematicians have also given a proof to the theorem but still non-explanatory. Incidentally, the article containing Conne's algebraic proof of the theorem, which won him a prize too, is translated by the author into Farsi and with some added comments and it is published in the Farhang Va Andisheh Riaz, a mathematical magazine in Farsi, see [1]. Karamzadeh, in three articles in 2014, 2015, and 2018, revisited the theorem (apparently, motivated by Conway's proof, see the latter article) and successfully resolved the possible mystery in the theorem after nearly a century, by stating the theorem in the form of "if, and only if", see [2–4]. Karamzadeh is the only mathematician, in the history of Morley's Theorem, who presented the theorem in this form, see [4]. And in his 2018 article, see [4], has provided some evidence that no simpler proof of the theorem, than the one given in his 2015 article, can ever be found in the future (note, the latter's elegant short article is a joint work collaborated with his previous student in mathematics education, the late I. Gorjian, and his colleague M. Namdari). The work of these three papers has recently been discussed, in detail, in the book "Mathematics and explanation" in [5]. I cannot help recalling our last meeting in Shiraz, which occurred a few weeks ago. While he was giving a talk at Shiraz university of technology, where my colleagues Azarpanah, and Namdari from Ahvaz were also present, he was talking enthusiastically about general mathematics in the same way as he was talking to us in that ice cream shop, nearly sixty years ago, except this time he did not treat us, but Dr. Fakharzadeh, who was the organizer of that meeting, did instead. Incidentally, we three were invited by Mr. Habibpour an Ex-student of Karamzadeh, who on the occasion of teachers' day, had arranged a private festival with 400 invitees to commemorate four of his previous school teachers, including Karamzadeh, who was his university teacher. It was a very interesting festival and we all enjoyed it very much. By the way, I also was given an opportunity in that festival to recall the above discussed problems, especially the coins problem, with Karamzadeh.

My second meeting with Karamzadeh was in 1352 when I was graduated by the university of Tehran, in 1348 I was accepted by the university of Tehran to study mathematics. Karamzadeh just completed his B. Sc. degree and was preparing to continue his studies towards the degree of Ph.D. in England. Me and Karamzadeh grown up in the city of Masjed Solayman and I was chasing him for a gap of four years. In our second meeting in the university of Tehran I asked him to write me a letter from his university in England so that I could have correspondence with him and he accepted. This is an excellent characteristic of him to absorb people as much as possible. Anyway during the years of my study at Tehran university I wrote some letters to him and discussed mathematics.

Incidentally when I completed my study at Tehran university I was awarded a scholarship to continue my study towards a Ph.D. degree. I chose to go to the university of Birmingham in England and this was due to the guidance of my friend prof. J. Moori, a friend of Karamzadeh, who was a Ph.D. student of that university at that time. Moori knew the home address of Karamzadeh and offered me to arrange a meeting with him in the city of Exeter. We went to this city and stayed at his house for a night. Karamzadeh just finished his Ph.D. studies at the university of Exeter and was employed by the university of Jundishapur in Ahvaz.

My third meeting with Karamzadeh was in 1357 when after finishing my Ph.D. studies and returned to Iran. Upon the recommendation of Karamzadeh, who was the dean of faculty of mathematics, statistics and computer science of the Jundishapur university, I was employed by this university starting from Farvardin 1357. We used to discuss mathematics from time to time. He told me that when in England talked to his Ph. D. thesis supervisor professor D. Rees about a subject of research he asked him to agree to work on a topic in non-commutative ring theory. Karamzadeh continued to say that this was not a wisely decision because prof. Rees was a pioneer of commutative ring theory in England and asking him for a research topic in non-commutative ring theory was non-sense. Anyway Karamzadeh said Rees told him that he is not familiar with non-commutative ring theory but if he feels comfortable with the subject it is ok but he cannot help me. Karamzadeh told me this was one the strange behavior of him that working on non-commutative ring theory under the supervision of a giant of commutative ring theory.

My stay at the Jundishapur university did not last long and in 1368 I joined the mathematics department of the Tehran university. My task at this university was to enhance the postgraduate program in mathematics which was successful. While I was in Ahvaz the war with Iraq started in Shahrivar 1359, but me and Karamzadeh and some other colleagues stayed in Ahvaz. Karamzadeh while working in algebra, he also continued his research on a different subject namely the rings of continuous functions called  $C(X)$ . To be precise let  $X$  be a topological space and consider the set of all continuous functions from  $X$  to the set of real numbers  $\mathbb{R}$ , this set is denoted by  $C(X)$  and under the point-wise addition and multiplication is a commutative ring. Karamzadeh found  $C(X)$  interesting and started to research on this ring and encouraged Ph. D. students to do their thesis about properties of  $C(X)$ . Some of the published papers are about the algebraic properties of the ring  $C(X)$  and the topological space  $X$  which are interesting and attracted the attention of experts. Incidentally,  $X$  is always assumed to be a Tychonoff space in these papers, for it is well-known that one, without any loss, can make this assumption.

At the end of this note I admire the efforts of Karamzadeh to upgrade the knowledge of faculties at the mathematics department of the Shahid Chamran university.

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